Frank Systems of Coherent Electrical Units ... More

In any coherent system TLMQ of fundamental time T, length L, mass M and charge Q

$$1\,\text{Derived Unit of Resistance} \triangleq \frac{1\,\text{Derived Unit of Action}}{1\,\text{Fundamental Unit of Charge}^2} = 1\,\mathrm{Q^{-2}{\cdot}M{\cdot}L^2{\cdot}T^{-1}}$$

When the unit of charge is chosen so that $Z_0 = 1\,\text{Derived Unit of Resistance} = 1\,\mathrm{Q^{-2}{\cdot}M{\cdot}L^2{\cdot}T^{-1}}$ then $1\,\mathrm{Q}$ is effectively a derived unit while maintaining the form of a fundamental unit — this leads to straightforward conversions to and from other systems of electromagnetic units. Formally setting $Z_0 = 1$ then leads to an equivalent but incommensurable TLM($Z_0 \triangleq 1$) system with unitless resistance, $\mu_0 = \varepsilon_0 = {}^1\!/_c$, a derived unit of charge $1\,\mathrm{M^{1/2}{\cdot}L{\cdot}T^{-1/2}} = \sqrt{1\,\text{Derived Unit of Action}}$, and having a streamlined presentation of the Ampère and Coulomb Laws.

Given a precursor system of units TLM★Q where the vacuum impedance Z_0 does not evaluate to $1\,{}^\star\mathrm{Q^{-2}{\cdot}M{\cdot}L^2{\cdot}T^{-1}}$ there is a commensurable unit of charge $1\,\mathrm{Q}$ that can be computed directly

$$1\,\mathrm{Q} \triangleq \sqrt{\frac{1\,\text{Derived Unit of Action}}{Z_0}} \equiv \sqrt{\frac{1\,\mathrm{M{\cdot}L^2{\cdot}T^{-1}}}{Z_0}} \quad \text{or equivalently} \quad \frac{1\,\mathrm{Q}}{1\,{}^\star\mathrm{Q}} \overset{\text{unitless}}{=} \sqrt{\frac{1\,{}^\star\mathrm{Q^{-2}{\cdot}M{\cdot}L^2{\cdot}T^{-1}}}{Z_0}}$$

That is to say, letting ζ_0 be the numerical value of the constant Z_0 in the T, L, M and ★Q units

$$\zeta_0 \triangleq \frac{Z_0}{1\,{}^\star\mathrm{Q^{-2}{\cdot}M{\cdot}L^2{\cdot}T^{-1}}}$$

gives the conversion

$$\frac{1\,\mathrm{Q}}{1\,{}^\star\mathrm{Q}} = \frac{1}{\sqrt{\zeta_0}} \quad \Longrightarrow \quad \frac{Z_0}{1\,\mathrm{Q^{-2}{\cdot}M{\cdot}L^2{\cdot}T^{-1}}} = 1$$

and a fundamental unit of charge $1\,\mathrm{Q}$ corresponding to the derived unit of charge $1\,\mathrm{M^{1/2}{\cdot}L{\cdot}T^{-1/2}}$.

The *frankcoulomb* FC — what the SI unit of charge *should have been* — is the fundamental unit of charge (via an exact conversion prior to the quantization implicit in the 2019 rebasing)

$$1\,\mathrm{FC} = \sqrt{\frac{1\,\mathrm{J{\cdot}s}}{Z_0}} = \frac{1}{\sqrt{4\pi \times 29.979\,245\,8}}\,\mathrm{C}$$

for which $Z_0 = 1\,{}^{\mathrm{J{\cdot}s}}\!/_{\mathrm{FC^2}}$ and

$$1\,\mathrm{kg^{1/2}{\cdot}m{\cdot}s^{-1/2}} = \sqrt{1\,\mathrm{J{\cdot}s}}$$

is the equivalent derived unit.

Other coherent electrical units are derived with respect to this unit via proportions

$$\begin{aligned}
\bullet^{-2} &= 4\pi \times 29.979\,245\,8 \approx 377\\
\bullet^{-1} &= \sqrt{4\pi \times 29.979\,245\,8} \approx 19.4\\
\bullet &= \frac{1}{\sqrt{4\pi\times 29.979\,245\,8}} \approx \frac{1}{19.4} \approx 51.5 \times 10^{-3}\\
\bullet^{2} &= \frac{1}{4\pi\times 29.979\,245\,8} \approx \frac{1}{377} \approx 2.65 \times 10^{-3}
\end{aligned}$$

which convert SI to the SMKF — *second-metre-kilogramme-frankcoulomb* — system or the equivalent but incommensurable SMK($Z_0 \triangleq 1$) system according to the power of Q in the unit's TLMQ dimensionality. Having done so, the corresponding unit in any other frank or TLM($Z_0 \triangleq 1$) system can be determined using only conversions appropriate for the unit's TLM dimensionality.

measurement		frank unit	conversion		$\propto$	TLMQ	TLM ($Z_0 \triangleq 1$)
electric charge	Q	frankcoulomb	FC	51.5 mC	$\bullet$	Q	$\mathsf{M}^{1/2}\mathsf{L}\mathsf{T}^{-1/2}$
electric current	I	frankamp	FA	51.5 mA	$\bullet$	$\mathsf{Q}\mathsf{T}^{-1}$	$\mathsf{M}^{1/2}\mathsf{L}\mathsf{T}^{-3/2}$
voltage	V	frankvolt	FV	19.4 V	$\bullet^{-1}$	$\mathsf{Q}^{-1}\mathsf{M}\mathsf{L}^{2}\mathsf{T}^{-2}$	
electric flux	Φ_E		FV·m	19.4 V·m	$\bullet^{-1}$	$\mathsf{Q}^{-1}\mathsf{M}\mathsf{L}^{3}\mathsf{T}^{-2}$	$\mathsf{M}^{1/2}\mathsf{L}^{2}\mathsf{T}^{-3/2}$
magnetic flux	Φ_B	frankweber	FWb	19.4 Wb	$\bullet^{-1}$	$\mathsf{Q}^{-1}\mathsf{M}\mathsf{L}^{2}\mathsf{T}^{-1}$	$\mathsf{M}^{1/2}\mathsf{L}\mathsf{T}^{-1/2}$
capacitance	C	frankfarad	FF	2.65 mF	$\bullet^{2}$	$\mathsf{Q}^{2}\mathsf{M}^{-1}\mathsf{L}^{-2}\mathsf{T}^{2}$	T
conductance	G	franksiemens	FS	2.65 mS	$\bullet^{2}$	$\mathsf{Q}^{2}\mathsf{M}^{-1}\mathsf{L}^{-2}\mathsf{T}$	1
conductivity	σ		FS/m	2.65 mS/m	$\bullet^{2}$	$\mathsf{Q}^{2}\mathsf{M}^{-1}\mathsf{L}^{-3}\mathsf{T}$	L^{-1}
resistance	R	frankohm $*Z_0{=}1$	FΩ	377 Ω	$\bullet^{-2}$	$\mathsf{Q}^{-2}\mathsf{M}\mathsf{L}^{2}\mathsf{T}^{-1}$	1
inductance	L	frankhenry	FH	377 H	$\bullet^{-2}$	$\mathsf{Q}^{-2}\mathsf{M}\mathsf{L}^{2}$	T
displacement field	D		FC/m²	51.5 mC/m²	$\bullet$	$\mathsf{Q}\mathsf{L}^{-2}$	$\mathsf{M}^{1/2}\mathsf{L}^{-1}\mathsf{T}^{-1/2}$
magnetizing field	H		FA/m	51.5 mA/m	$\bullet$	$\mathsf{Q}\mathsf{L}^{-1}\mathsf{T}^{-1}$	$\mathsf{M}^{1/2}\mathsf{T}^{-3/2}$
electric field	E		FV/m	19.4 V/m	$\bullet^{-1}$	$\mathsf{Q}^{-1}\mathsf{M}\mathsf{L}\mathsf{T}^{-2}$	
magnetic field	B	franktesla	FT	19.4 T	$\bullet^{-1}$	$\mathsf{Q}^{-1}\mathsf{M}\mathsf{T}^{-1}$	$\mathsf{M}^{1/2}\mathsf{L}^{-1}\mathsf{T}^{-1/2}$
permittivity	ε	$*\varepsilon_0{=}1/299\,792\,458$	FF/m	2.65 mF/m	$\bullet^{2}$	$\mathsf{Q}^{2}\mathsf{M}^{-1}\mathsf{L}^{-3}\mathsf{T}^{2}$	$\mathsf{L}^{-1}\mathsf{T}$
permeability	μ	$*\mu_0{=}1/299\,792\,458$	FH/m	377 H/m	$\bullet^{-2}$	$\mathsf{Q}^{-2}\mathsf{M}\mathsf{L}$	

Note that the franksiemens and frankohm, being directly derived from the impedance of free space, are invariant across all systems of coherent TLM units. And the frankhenry and frankfarad are common to all coherent systems of units based on the second.

The Derived Units Corresponding to the Frank Units are Unexpected

measurement		frank unit	SMKF		SMK($Z_0 \triangleq 1$)	SI
electric charge	Q	frankcoulomb	FC	FC	$\sqrt{\mathrm{J{\cdot}s}}$	51.5 mC
electric current	I	frankamp	FA	FC/s	$\sqrt{\mathrm{W}}$	51.5 mA
voltage	V	frankvolt	FV	J/FC		19.4 V
electric flux	Φ_E		FV·m	J·m/FC	$\sqrt{\mathrm{W}}{\cdot}\mathrm{m}$	19.4 V·m
magnetic flux	Φ_B	frankweber	FWb	J·s/FC	$\sqrt{\mathrm{J{\cdot}s}}$	19.4 Wb
capacitance	C	frankfarad	FF	FC/FV	s	2.65 mF
conductance	G	franksiemens	FS	FA/FV	1	2.65 mS
conductivity	σ		FS/m	FA/FV·m	m^{-1}	2.65 mS/m
resistance	R	frankohm $*Z_0{=}1$	FΩ	FV/FA	1	377 Ω
inductance	L	frankhenry	FH	FWb/FA	s	377 H
displacement fd.	D		FC/m²	FC/m²	$\sqrt{\mathrm{J{\cdot}s}}/\mathrm{m}^2$	51.5 mC/m²
magnetizing fd.	H		FA/m	FA/m	$\sqrt{\mathrm{W}}/\mathrm{m}$	51.5 mA/m
electric field	E		FV/m	FV/m		19.4 V/m
magnetic field	B	franktesla	FT	FWb/m²	$\sqrt{\mathrm{J{\cdot}s}}/\mathrm{m}^2$	19.4 T
permittivity	ε	$*\varepsilon_0{=}1/299\,792\,458$	FF/m	FC/FV·m	s/m	2.65 mF/m
permeability	μ	$*\mu_0{=}1/299\,792\,458$	FH/m	FWb/FA·m		377 H/m