Handicapping Mental Arithmetic Primer

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A General Purpose Handicap for Time-on-Distance and Time-on-Time

Pace is a measure of how many seconds it takes to complete a mile and varies inversely to speed measured in knots. So, for example, an average speed of 6 knots corresponds to a pace of 600 $^{\text{seconds}}/_{\text{mile}}$ and an average speed of 4 knots corresponds to a pace of 900 $^{\text{seconds}}/_{\text{mile}}$ — pace and speed multiplied together always results in 3600 $^{\text{seconds}}/_{\text{hour}}$. Pace is the natural measure of performance prediction and handicapping; inverting a pace to get a speed can be instructive but is never actually needed.

A general purpose handicap g is a boat's pace on average and can be used for either time-on-distance or time-on-time handicapping. A slower pace is represented by a greater number of $^{\text{seconds}}/_{\text{mile}}$ and the following example boats are ordered from fastest to slowest. The "delta-gee" Δg column shows the differences in handicap from our own boat, *Shindig*, which we identify with a circle $\circ$. Units are not shown in the table but handicaps are understood to be $^{\text{seconds}}/_{\text{mile}}$ with $^{\text{minutes and seconds}}/_{\text{mile}}$ in brackets (formatted to be mm:ss); the inverse g^{-1} column is in knots. These handicaps are rounded to the closest multiple of 3 $^{\text{s}}/_{\text{mile}}$ so it will be natural to reckon time allowances in unit thirds.

Example Boat	g	Δg	g^{-1}	Make
Hurricane	729(12:09)	−132(2:12)	4.938	Buddy 24
Winged Elephant	810(13:30)	−51	4.444	Frequency 24
Mechanical Drone	834(13:54)	−27	4.317	See in Sea 30
Shindig	861(14:21)	$\circ$	4.181	Raider 28
Professor	864(14:24)	+3	4.167	Stone 22
Rhumb Punch	876(14:36)	+15	4.110	Chimera 33

A PHRF rating (or any time-on-distance rating system with handicaps in units of seconds per mile) is the difference in general purpose handicap from that of the zero-rated boat. Should the zero-rated boat have as its general purpose handicap ${}^{\text{zero-rated}}g = 600\,{}^{\text{seconds}}/_{\text{mile}}$ (for example) then adding 600 $^{\text{s}}/_{\text{mile}}$ to our own boat's PHRF rating will recover our boat's general purpose handicap. Further note that a difference in general purpose handicaps is equal to the corresponding difference in PHRF ratings

$$\Delta g = \Delta\text{PHRF}$$

It turns out that all we need to compare ourselves to our competitors on the water is our own boat's general purpose handicap together with the "delta" in PHRF ratings.

A sensible PHRF station interested in pursuing time-on-time handicapping will simply publish its value for ${}^{\text{zero-rated}}g$. A misguided PHRF station may publish a transformation formula (with named parameters "A" and "B") to convert a time-on-distance based PHRF rating to a time-on-time based TCF (time correction factor)

$$\mathrm{TCF} = \frac{\mathrm{A}}{\mathrm{B} + \mathrm{PHRF}}$$

The "A" parameter in the numerator is irrelevant. The "B" parameter in the denominator is the needed ${}^{\text{zero-rated}}g$. Be assured that this transformation is completely superfluous — boats will handicap exactly the same as described here.

Note, we are using the italic type together with a greek uppercase 'D' to visually identify the one character g, d, t and p and two character Δg, Δt and Δp named variables. Being short and lacking specificity, these require accompanying text to explain what they represent. Most handicapping literature uses typewriter-friendly multi-character uppercase roman-type variable names GPH, CL, ET, ETPM, DGPH, TA, and TAPM that are supposedly self-explanatory!?

From our perspective a time allowance "delta-tee" Δt is the time ahead or behind us that a competing boat must finish in order to tie with us after handicapping is applied. Likewise a pace allowance "delta-pee" Δp is a difference in course-average pace necessary for a tie. Multiplication by course length d connects our course-average pace p to our elapsed time t and a pace allowance Δp to a time allowance Δt.

- For time-on-distance handicapping the pace allowance for a competitor Δp is fixed at Δg. By multiplying with the known course length, the time allowance is predetermined and wont vary however long it takes us to finish the course.
- For time-on-time handicapping the relationship between our observed course-average pace p and the pace allowance Δp is best expressed as a proportionality. The ratio of Δp to Δg is equal in proportion to the ratio of p to g. Time-on-time handicapping is independent of course length so turning a pace allowance into a time allowance can be achieved by simply dropping per-mile from all the units in the proportionality. We'll show this more thoroughly in the worked examples below.

Our boat *Shindig* has the handicap $g = 861\,{}^{\mathrm{s}}\!/\!_{\mathrm{mile}} = {}^{14\,\mathrm{min}\,21\,\mathrm{s}}\!/\!_{\mathrm{mile}}$. On average Shindig should take $861\,\mathrm{s} = 14\,\mathrm{min}\,21\,\mathrm{s}$ to complete a mile of the course or, in thirds, $287\,\mathrm{s} = 4\,\mathrm{min}\,47\,\mathrm{s}$ to complete a third of a mile. If a race course were four and one-third miles long we would add the expected elapsed time on a four mile course to that on a third of a mile course. Using the "varies in proportion to" $\propto$ notation

$$\begin{array}{rrcl} & 57\,\mathrm{min}\,24\,\mathrm{s} & \propto & 4\,\mathrm{mile} \\ + & 4\,\mathrm{min}\,47\,\mathrm{s} & \propto & {}^{1}\!/\!_{3}\,\mathrm{mile} \\ \hline & 62\,\mathrm{min}\,11\,\mathrm{s} & \propto & 4^{1}\!/\!_{3}\,\mathrm{mile} \end{array} \left\{ \begin{array}{rcll} t & \propto & d \text{ (on average)} & \\ 14\,\mathrm{min}\,21\,\mathrm{s} & \propto & 1\,\mathrm{mile} & \\ \hline 28\,\mathrm{min}\,42\,\mathrm{s} & \propto & 2\,\mathrm{mile} & (2\times) \\ 43\,\mathrm{min}\ \ 3\,\mathrm{s} & \propto & 3\,\mathrm{mile} & (3\times) \\ 57\,\mathrm{min}\,24\,\mathrm{s} & \propto & 4\,\mathrm{mile} & (4\times) \\ 1\,\mathrm{h}\,11\,\mathrm{min}\,45\,\mathrm{s} & \propto & 5\,\mathrm{mile} & (5\times) \\ & \vdots & & \\ \hline 4\,\mathrm{min}\,47\,\mathrm{s} & \propto & {}^{1}\!/\!_{3}\,\mathrm{mile} & ({}^{1}\!/\!_{3}\times) \\ 9\,\mathrm{min}\,34\,\mathrm{s} & \propto & {}^{2}\!/\!_{3}\,\mathrm{mile} & ({}^{2}\!/\!_{3}\times) \end{array} \right.$$

Were we to finish this course with an elapsed time of 1 h 2 min 11 s then all the time allowances calculated with regard to time-on-time would be the same as for time-on-distance.

We demonstrated techniques of mental arithmetic in the working above. For example, we replaced a multiplication with sequential additions. Getting from 14 min 21 s to 28 min 42 s was easy, we simply doubled all the digits. To get to the multiple of three we add these. 28 min plus 14 min is 42 min. 42 s plus 21 s is 63 s giving us 42 min 63 s in all. The seconds overflowed so we reshuffle this to get 43 min 3 s. We always works from the big to the little (bigendian!) — we start with the largest effect and then refine it with further smaller adjustments to improve the accuracy of the result. If we can work out the largest effects beforehand and write them in a table we can save ourselves a lot of work — time allowances are well suited this. At the very least we should always have a table of differences in handicap and, if using time-on-time, a table of expected elapsed times at reasonable intervals.

Reckoning Time Allowances from Our Boat *Shindig's* Point of View

For our competitor *Rhumb Punch* the table of handicaps states that $\Delta g = 15\,{}^{\mathrm{s}}\!/\!_{\mathrm{mile}}$ (Or equivalently that $\Delta\mathrm{PHRF} = 15\,{}^{\mathrm{s}}\!/\!_{\mathrm{mile}}$). For time-on-distance handicapping every mile of course length contributes 15 s to the time allowance Δt. For each additional ⅓ mile the time allowance is increased by 5 s. On a four and one-third mile course this would yield a time allowance of 65 s

$$
\begin{array}{rrl}
 & 60\,\mathrm{s} \propto & 4\,\mathrm{mile} \\
+ & 5\,\mathrm{s} \propto & {}^{1}\!/\!_{3}\,\mathrm{mile} \\
\hline
 & 65\,\mathrm{s} \propto & 4{}^{1}\!/\!_{3}\,\mathrm{mile}
\end{array}
\quad
\left\{
\begin{array}{rll}
\Delta t \propto & d \text{ (time-on-distance)} & \\
15\,\mathrm{s} \propto & 1\,\mathrm{mile} & \\
\hline
30\,\mathrm{s} \propto & 2\,\mathrm{mile} & (2\times) \\
45\,\mathrm{s} \propto & 3\,\mathrm{mile} & (3\times) \\
60\,\mathrm{s} \propto & 4\,\mathrm{mile} & (4\times) \\
\hline
5\,\mathrm{s} \propto & {}^{1}\!/\!_{3}\,\mathrm{mile} & ({}^{1}\!/\!_{3}\times)
\end{array}
\right.
$$

For time-on-time handicapping the ratio of the time allowance Δt to 15 s is equal in proportion to the ratio of elapsed time t to 14 min 21 s. That is, for every 14 min 21 s of elapsed time t the time allowance Δt increases by 15 s. In unit thirds, for every 4 min 47 s of elapsed time the time allowance increases by 5 s. At an elapsed time of 1 h 2 min 11 s we would expect a 65 s time allowance, the same as for time-on-distance handicapping on a four and one-third mile course

$$
\begin{array}{rrl}
 & 60\,\mathrm{s} \propto & 57\,\mathrm{min}\,24\,\mathrm{s} \\
+ & 5\,\mathrm{s} \propto & 4\,\mathrm{min}\,47\,\mathrm{s} \\
\hline
 & 65\,\mathrm{s} \propto & 62\,\mathrm{min}\,11\,\mathrm{s}
\end{array}
\quad
\left\{
\begin{array}{rll}
\Delta t \propto & t \text{ (time-on-time)} & \\
15\,\mathrm{s} \propto & 14\,\mathrm{min}\,21\,\mathrm{s} & \\
\hline
30\,\mathrm{s} \propto & 28\,\mathrm{min}\,42\,\mathrm{s} & (2\times) \\
45\,\mathrm{s} \propto & 43\,\mathrm{min}\,\ \ 3\,\mathrm{s} & (3\times) \\
60\,\mathrm{s} \propto & 57\,\mathrm{min}\,24\,\mathrm{s} & (4\times) \\
\hline
5\,\mathrm{s} \propto & 4\,\mathrm{min}\,47\,\mathrm{s} & ({}^{1}\!/\!_{3}\times)
\end{array}
\right.
$$

The ratio of 65 s to 15 s is equal in proportion to the ratio of 62 min 11 s to 14 min 21 s. To reiterate: the ratio of the reckoned time allowance to the difference in handicaps (dropping per-mile from the unit) is equal in proportion to the ratio of our own elapsed time to our own general purpose handicap (dropping per-mile from the unit).

The overall pattern is obvious. On average Δt, t and d vary in lockstep

$$\begin{array}{cr} & 60\,\mathrm{s} \propto 57\,\mathrm{min}\,24\,\mathrm{s} \propto 4\,\mathrm{mile} \\ + & 5\,\mathrm{s} \propto 4\,\mathrm{min}\,47\,\mathrm{s} \propto \tfrac{1}{3}\,\mathrm{mile} \\ \hline & 65\,\mathrm{s} \propto 62\,\mathrm{min}\,11\,\mathrm{s} \propto 4\tfrac{1}{3}\,\mathrm{mile} \end{array} \quad \left\{ \begin{array}{rl} \Delta t \propto \quad t \quad \propto d & \\ 15\,\mathrm{s} \propto 14\,\mathrm{min}\,21\,\mathrm{s} \propto 1\,\mathrm{mile} & \\ \hline 30\,\mathrm{s} \propto 28\,\mathrm{min}\,42\,\mathrm{s} \propto 2\,\mathrm{mile} & (2\times) \\ 45\,\mathrm{s} \propto 43\,\mathrm{min}\,3\,\mathrm{s} \propto 3\,\mathrm{mile} & (3\times) \\ 60\,\mathrm{s} \propto 57\,\mathrm{min}\,24\,\mathrm{s} \propto 4\,\mathrm{mile} & (4\times) \\ \hline 5\,\mathrm{s} \propto 4\,\mathrm{min}\,47\,\mathrm{s} \propto \tfrac{1}{3}\,\mathrm{mile} & (\tfrac{1}{3}\times) \end{array} \right.$$

For an actual race which departs from the average, time allowances are dependent on either time or distance depending on the style of handicapping. Were we to take exactly one hour to finish a race using time-on-time handicapping, the time allowance for 57 min 24 s would fall short and the time allowance for 62 min 11 s would overshoot. But we only need about two and half minutes worth of additional time allowance to round out the 57 min 24 s worth. As a rough estimate every five minutes of elapsed time increases the time allowance by five seconds. So $2.5\,\mathrm{s} \propto 2.5\,\mathrm{min}$, approximately. This would give a total time allowance of about 62.5 s. To be certain of the win, we must cross the finish line at least 1 min 3 s before Rhumb Punch.

For our competitor *Professor* we have $\Delta g = 3\,{}^{\mathrm{s}}\!/_{\mathrm{mile}}$ from the table of handicaps. On the same four and one-third mile course with time-on-distance handicapping or taking the same 62 min 11 s with time-on-time handicapping

$$\begin{array}{cr} & 12\,\mathrm{s} \propto 57\,\mathrm{min}\,24\,\mathrm{s} \propto 4\,\mathrm{mile} \\ + & 1\,\mathrm{s} \propto 4\,\mathrm{min}\,47\,\mathrm{s} \propto \tfrac{1}{3}\,\mathrm{mile} \\ \hline & 13\,\mathrm{s} \propto 62\,\mathrm{min}\,11\,\mathrm{s} \propto 4\tfrac{1}{3}\,\mathrm{mile} \end{array} \quad \left\{ \begin{array}{r} \Delta t \propto \quad t \quad \propto d \\ 3\,\mathrm{s} \propto 14\,\mathrm{min}\,21\,\mathrm{s} \propto 1\,\mathrm{mile} \\ \hline 6\,\mathrm{s} \propto 28\,\mathrm{min}\,42\,\mathrm{s} \propto 2\,\mathrm{mile} \\ 9\,\mathrm{s} \propto 43\,\mathrm{min}\,3\,\mathrm{s} \propto 3\,\mathrm{mile} \\ 12\,\mathrm{s} \propto 57\,\mathrm{min}\,24\,\mathrm{s} \propto 4\,\mathrm{mile} \\ \hline 1\,\mathrm{s} \propto 4\,\mathrm{min}\,47\,\mathrm{s} \propto \tfrac{1}{3}\,\mathrm{mile} \end{array} \right.$$

For every mile of distance or for every 14 min 21 s of elapsed time, the time allowance we must give Professor increases by 3 s. Likewise, For every third of a mile or 4 min 47 s the time allowance increases by 1 s. We can repeat this with the Δg for each of our competitors to describe all the time allowances we need.

For competitors we have seen so far, adding the superscript $^{\mathrm{Prof}}$ for Professor and $^{\mathrm{RP}}$ for Rhumb Punch

$$\begin{array}{cr} & 12\,\mathrm{s} \propto 60\,\mathrm{s} \propto 57\,\mathrm{min}\,24\,\mathrm{s} \propto 4\,\mathrm{mile} \\ + & 1\,\mathrm{s} \propto 5\,\mathrm{s} \propto 4\,\mathrm{min}\,47\,\mathrm{s} \propto \tfrac{1}{3}\,\mathrm{mile} \\ \hline & 13\,\mathrm{s} \propto 65\,\mathrm{s} \propto 62\,\mathrm{min}\,11\,\mathrm{s} \propto 4\tfrac{1}{3}\,\mathrm{mile} \end{array} \quad \left\{ \begin{array}{r} \overset{\mathrm{Prof}}{\Delta t} \propto \overset{\mathrm{RP}}{\Delta t} \propto \quad t \quad \propto d \\ 3\,\mathrm{s} \propto 15\,\mathrm{s} \propto 14\,\mathrm{min}\,21\,\mathrm{s} \propto 1\,\mathrm{mile} \\ \hline 6\,\mathrm{s} \propto 30\,\mathrm{s} \propto 28\,\mathrm{min}\,42\,\mathrm{s} \propto 2\,\mathrm{mile} \\ 9\,\mathrm{s} \propto 45\,\mathrm{s} \propto 43\,\mathrm{min}\,3\,\mathrm{s} \propto 3\,\mathrm{mile} \\ 12\,\mathrm{s} \propto 60\,\mathrm{s} \propto 57\,\mathrm{min}\,24\,\mathrm{s} \propto 4\,\mathrm{mile} \\ \hline 1\,\mathrm{s} \propto 5\,\mathrm{s} \propto 4\,\mathrm{min}\,47\,\mathrm{s} \propto \tfrac{1}{3}\,\mathrm{mile} \end{array} \right.$$

To summarize, for each of our competitors Δg or ΔPHRF ($\times 1$ mile) is the difference in handicap dropping per-mile from the unit: in the time-on-distance case each mile of the course contributes this to the time allowance for the corresponding boat; whereas in the time-on-time case each 14 min 21 s of our own elapsed time contributes this to the time allowance. Here 14 min 21 s is just our own general purpose handicap g dropping per-mile from the unit ($\times 1$ mile).

The table of handicaps also gives us $\overset{\text{MD}}{\Delta g} = -27\,{}^{\text{s}}/_{\text{mile}}$, $\overset{\text{WE}}{\Delta g} = -51\,{}^{\text{s}}/_{\text{mile}}$ and $\overset{\text{Hurr}}{\Delta g} = -132\,{}^{\text{s}}/_{\text{mile}}$ for our competitors *Mechanical Drone*, *Winged Elephant* and *Hurricane* respectively. The negative sign simply means the time allowance is in our favour — we will drop the sign (with a little finesse) in the presentation below. When expressing variations in proportion over multiple boats it is more conventional to write the distance and time on the left and the have per-boat time allowances on the right, where we order competitors by the magnitude of Δg (and using superscripts on the variables to identify competitors)

$$
\begin{array}{rcrcrcrcrcrcrcr}
d & \propto & t & \propto & \overset{\text{Prof}}{\Delta t} & \propto & \overset{\text{RP}}{\Delta t} & \propto & -\overset{\text{MD}}{\Delta t} & \propto & -\overset{\text{WE}}{\Delta t} & \propto & -\overset{\text{Hurr}}{\Delta t} \\
1\,\text{mile} & \propto & 14\,\text{min}\,21\,\text{s} & \propto & 3\,\text{s} & \propto & 15\,\text{s} & \propto & 27\,\text{s} & \propto & 51\,\text{s} & \propto & 132\,\text{s} \\
\hline
2\,\text{mile} & \propto & 28\,\text{min}\,42\,\text{s} & \propto & 6\,\text{s} & \propto & 30\,\text{s} & \propto & 54\,\text{s} & \propto & 102\,\text{s} & \propto & 264\,\text{s} \\
3\,\text{mile} & \propto & 43\,\text{min}\ \ 3\,\text{s} & \propto & 9\,\text{s} & \propto & 45\,\text{s} & \propto & 81\,\text{s} & \propto & 153\,\text{s} & \propto & 396\,\text{s} \\
4\,\text{mile} & \propto & 57\,\text{min}\,24\,\text{s} & \propto & 12\,\text{s} & \propto & 60\,\text{s} & \propto & 108\,\text{s} & \propto & 204\,\text{s} & \propto & 528\,\text{s} \\
\hline
{}^1\!/_3\,\text{mile} & \propto & 4\,\text{min}\,47\,\text{s} & \propto & 1\,\text{s} & \propto & 5\,\text{s} & \propto & 9\,\text{s} & \propto & 17\,\text{s} & \propto & 44\,\text{s}
\end{array}
$$

This presentation is mathematically precise but visually cluttered. Proportions are highly suited to being expressed in a table; whereas, the above notation is best suited for annotating additions.

$$
\begin{array}{crcrcrcrcrcrcr}
 & 4\,\text{mile} & \propto & 57\,\text{min}\,24\,\text{s} & \propto & 12\,\text{s} & \propto & 60\,\text{s} & \propto & 108\,\text{s} & \propto & 204\,\text{s} & \propto & 528\,\text{s} \\
+ & {}^1\!/_3\,\text{mile} & \propto & 4\,\text{min}\,47\,\text{s} & \propto & 1\,\text{s} & \propto & 5\,\text{s} & \propto & 9\,\text{s} & \propto & 17\,\text{s} & \propto & 44\,\text{s} \\
\hline
 & 4{}^1\!/_3\,\text{mile} & \propto & 62\,\text{min}\,11\,\text{s} & \propto & 13\,\text{s} & \propto & 65\,\text{s} & \propto & 117\,\text{s} & \propto & 221\,\text{s} & \propto & 572\,\text{s}
\end{array}
$$

In tables of time allowances we cut down on visual clutter by just expressing the values, omitting units, uniformly expressing intervals of time as hours:minutes:seconds and by ordering results to make them easier to look up.

Shindig	14:21
Professor	+3
Rhumb Punch	+15
Mechanical Drone	−27
Winged Elephant	−51
Hurricane	−2:12

We would do well to fill out the table by adding an entry for ⅔ of a mile as well as entries for 5 through 9 miles.

Shindig		+Prof	+RP	−MD	−WE	−Hurr
⅓	4:47	1	5	9	17	44
⅔	9:34	2	10	18	34	1:28
1	14:21	3	15	27	51	2:12
2	28:42	6	30	54	1:42	4:24
3	43:03	9	45	1:21	2:33	6:36
4	57:24	12	1:00	1:48	3:24	8:48
5	1:11:45	15	1:15	2:15	4:15	11:00
6	1:26:06	18	1:30	2:42	5:06	13:12
7	1:40:27	21	1:45	3:09	5:57	15:24
8	1:54:48	24	2:00	3:36	6:48	17:36
9	2:09:09	27	2:15	4:03	7:39	19:48

Even better, by expanding the vertical scale a bit (using as much height as fits the space available), we have a very easy to read table, more than sufficient for any around-the-buoys race with these five competitors.

Finally, we will also add a legend for the abbreviations used in the column headings.

∘	Shindig	14:21
Prof	Professor	+3
RP	Rhumb Punch	+15
MD	Mechanical Drone	−27
WE	Winged Elephant	−51
Hurr	Hurricane	−2:12

A table should be prepared prior to racing and need not be tabulated by hand; indeed, a conscientious race committee would prepare such a table for us. These tables express exact proportions, but in practice we must interpolate between lines to approximate the final time allowance.

The full table can help us with interpolation by refining our proportionality — minutes $\propto$ hours:minutes translates directly to seconds $\propto$ minutes:seconds. For example with Winged Elephant we can look down the column to notice that 1 min 59 s $\propto$ 33 min 29 s from which we get the excellent approximation of 2 s $\propto$ 33.5 s. So at elapsed time of 1 h 8 min we look at the table under 4⅔ mile to get a time allowance of 3 min 58 s at 1 h 6 min 58 s. We need another minute of elapsed time which is approximately another 4 s of time allowance to give a final time allowance of 4 min 2 s.

Shindig		+Prof	+RP	−MD	−WE	−Hurr
1/3	4:47	1	5	9	17	44
2/3	9:34	2	10	18	34	1:28
1	14:21	3	15	27	51	2:12
1 1/3	19:08	4	20	36	1:08	2:56
1 2/3	23:55	5	25	45	1:25	3:40
2	28:42	6	30	54	1:42	4:24
2 1/3	33:29	7	35	1:03	1:59	5:08
2 2/3	38:16	8	40	1:12	2:16	5:52
3	43:03	9	45	1:21	2:33	6:36
3 1/3	47:50	10	50	1:30	2:50	7:20
3 2/3	52:37	11	55	1:39	3:07	8:04
4	57:24	12	1:00	1:48	3:24	8:48
4 1/3	1:02:11	13	1:05	1:57	3:41	9:32
4 2/3	1:06:58	14	1:10	2:06	3:58	10:16
5	1:11:45	15	1:15	2:15	4:15	11:00
5 1/3	1:16:32	16	1:20	2:24	4:32	11:44
5 2/3	1:21:19	17	1:25	2:33	4:49	12:28
6	1:26:06	18	1:30	2:42	5:06	13:12
6 1/3	1:30:53	19	1:35	2:51	5:23	13:56
6 2/3	1:35:40	20	1:40	3:00	5:40	14:40
7	1:40:27	21	1:45	3:09	5:57	15:24
7 1/3	1:45:14	22	1:50	3:18	6:14	16:08
7 2/3	1:50:01	23	1:55	3:27	6:31	16:52
8	1:54:48	24	2:00	3:36	6:48	17:36
8 1/3	1:59:35	25	2:05	3:45	7:05	18:20
8 2/3	2:04:22	26	2:10	3:54	7:22	19:04
9	2:09:09	27	2:15	4:03	7:39	19:48
9 1/3	2:13:56	28	2:20	4:12	7:56	20:32
9 2/3	2:18:43	29	2:25	4:21	8:13	21:16

For Mechanical Drone we can look down the table to get 1 min 3 s $\propto$ 33 min 29 s for an approximate 1 s $\propto$ 33.5 s or an even more precise 1 min 57 s $\propto$ 1 h 2 min 11 s for a better approximation 2 s $\propto$ 1 min 2 s or, best of all, 3 min $\propto$ 1 h 35 min 40 s for the approximation 3 s $\propto$ 1 min 36 s; this further simplifies to 1 s $\propto$ 32 s, more precise and simpler than our first approximation drawn from the table. At 1 h 8 min we would refine the 2 min 6 s time allowance at 1 h 6 min 58 s in the table with a further 2 s to get the very accurate time allowance of 2 min 8 s.

For Hurricane 8 min 4 s $\propto$ 52 min 37 s giving an excellent approximation of 8 s $\propto$ 52.5 s and a further refinement using a poorer approximation of 1 s $\propto$ 7 s. These aren't best rational approximations in the number-theoretic sense; but they are good-enough.

With these approximate proportionalities we can accurately work out time-on-time handicapping on the water. These approximations are also best worked out beforehand. They are easily gleaned from the fully worked out table but aren't really suited to be added to a table numerically.

Shindig	$\Delta t \propto t$ Approximations			
Professor	1 s ∝ 4 min 47 s	1 s ∝ 5 min		
Rhumb Punch	5 s ∝ 4 min 47 s	1 s ∝ 1 min (via 5 s ∝ 5 min)		
Mechanical Drone	9 s ∝ 4 min 47 s	1 s ∝ 32 s (via 3 s ∝ 1 min 36 s)		
Winged Elephant	17 s ∝ 4 min 47 s	4 s ∝ 1 min 7 s	2 s ∝ 33.5 s	1 s ∝ 17 s
Hurricane	44 s ∝ 4 min 47 s	11 s ∝ 1 min 12 s	8 s ∝ 52.5 s	1 s ∝ 7 s

Complementary Tables of Time Allowances

Consider three boats, any of which might be considered *our boat* for comparisons sake.

Boat		g	Δg	Δg	Δg	Make
Hurr	Hurricane	729(12:09)	◦	−105(1:45)	−132(2:12)	Buddy 24
MD	Mechanical Drone	834(13:54)	+105(1:45)	◦	−27	See in Sea 30
Shin	Shindig	861(14:21)	+132(2:12)	+27	◦	Raider 28

How do time allowances compare when taken from these different points of view? Unlike before, we are including the sign of the time allowance in the body of the table to highlight the complementary columns.

Hurricane	12:09
Mechanical Drone	+1:45
Shindig	+2:12

Mechanical Drone	13:54
Shindig	+27
Hurricane	−1:45

Shindig	14:21
Mechanical Drone	−27
Hurricane	−2:12

Hurr		MD	Shin
1/3	4:03	+35	+44
2/3	8:06	+1:10	+1:28
1	12:09	+1:45	+2:12
2	24:18	+3:30	+4:24
3	36:27	+5:15	+6:36
4	48:36	+7:00	+8:48
5	1:00:45	+8:45	+11:00
6	1:12:54	+10:30	+13:12
7	1:25:03	+12:15	+15:24
8	1:37:12	+14:00	+17:36
9	1:49:21	+15:45	+19:48

MD		Shin	Hurr
1/3	4:38	+9	−35
2/3	9:16	+18	−1:10
1	13:54	+27	−1:45
2	27:48	+54	−3:30
3	41:42	+1:21	−5:15
4	55:36	+1:48	−7:00
5	1:09:30	+2:15	−8:45
6	1:23:24	+2:42	−10:30
7	1:37:18	+3:09	−12:15
8	1:51:12	+3:36	−14:00
9	2:05:06	+4:03	−15:45

Shin		MD	Hurr
1/3	4:47	−9	−44
2/3	9:34	−18	−1:28
1	14:21	−27	−2:12
2	28:42	−54	−4:24
3	43:03	−1:21	−6:36
4	57:24	−1:48	−8:48
5	1:11:45	−2:15	−11:00
6	1:26:06	−2:42	−13:12
7	1:40:27	−3:09	−15:24
8	1:54:48	−3:36	−17:36
9	2:09:09	−4:03	−19:48

We can easily correlate the potential ties on corrected time across the tables; each boat's viewpoint is different but complementary; the time allowances taken from the tables are consistent between these points of view.

Weaknesses of Time Allowances

Time allowances and corrected times represent the same handicapping relationship between boats. One is not an approximation of the other — they are identical. We have stressed how to accurately approximate time allowances using mental arithmetic but there is no limit on how precise those reckonings might be. But if the race committee rounds corrected times — a terrible practice that hasn't yet been eradicated — a time allowance as calculated here will fail to distinguish the one second interval of time when a tie on rounded corrected time is possible.

Time allowances reckoned from our point of view will tell us how we stand against any one of our competitors but it can't determine how they stand against each other. Corrected times do present an overall ordering of all competitors but they are a terrible nuisance to work with. There are ways to use our table of time allowances to approximate how our competitors fare against each other but it is beyond the scope of this document.

In Conclusion

Reckoning good approximations using mental arithmetic is a talent best acquired through practice. The tables developed here should provide an excellent first step in acquiring that talent. Race committees should make a conscientious effort to see that such tables are available for entered boats — online registration and scoring systems should make this easy.