Deriving Natural Units

Derived vs. Fundamental Units in a System of Natural Units

In a natural system the difference between derived and fundamental units is a matter of degree; indeed, in a system of wholly derived units all measurements may be considered unitless.

<table>
<tr><th>system</th><th>time</th><th>length</th><th>speed</th><th>mass</th><th>impulse</th><th>force</th><th>work</th><th>action</th><th>temp.</th><th>charge</th></tr>
<tr><td>T-L-M-Θ-Q</td><td rowspan="4">T</td><td rowspan="2">L</td><td rowspan="2">LT^{-1}</td><td rowspan="3">M</td><td rowspan="2">MLT^{-1}</td><td rowspan="2">MLT^{-2}</td><td rowspan="2">ML^2T^{-2}</td><td rowspan="2">ML^2T^{-1}</td><td>Θ</td><td>Q</td></tr>
<tr><td>T-L-M</td><td>ML^2T^{-2}</td><td>$M^{1/2}LT^{-1/2}$</td></tr>
<tr><td>T-M</td><td rowspan="2">T</td><td rowspan="2">1</td><td>M</td><td>MT^{-1}</td><td>M</td><td>MT</td><td>M</td><td>$M^{1/2}T^{1/2}$</td></tr>
<tr><td>T</td><td>T^{-1}</td><td>T^{-1}</td><td>T^{-2}</td><td>T^{-1}</td><td>1</td><td>T^{-1}</td><td>1</td></tr>
</table>

Within a term, units will be written in a reverse fashion with the more fundamental on the right and the more derived on the left. For our natural units we will set aside a few useful names and abbreviations based on a triple of (common name abbrev. / system of units abbrev. / dimensionality abbrev.)

time [T]	length [L]	mass [M]	force	work	temp. [Θ]	charge [Q]
s•t neosecond	f•l neofoot	q•m quatermass	dyn•f neodyne	erg•e neoerg	°•a neodegree	F•q neofranklin

Base for Time Derived Units (given an arbitrary fundamental time T)

$$\begin{array}{rl} Z_0 &= 1\,Q^{-2}ML^2T^{-1} \\ k &= 1\,\Theta^{-1}ML^2T^{-2} \\ \hbar &= 1\,ML^2T^{-1} \\ c &= 1\,LT^{-1} \\ T &= 1\,T \end{array} \Leftrightarrow \left.\begin{array}{rl} 1\,Q &= \sqrt{\hbar/Z_0} \\ 1\,\Theta &= \frac{\hbar}{kT} \\ 1\,M &= \frac{\hbar}{c^2T} \\ 1\,L &= cT \\ 1\,T &= T \end{array}\right\}\left\{\begin{bmatrix} -2 & 0 & 1 & 2 & -1 \\ & -1 & 1 & 2 & -2 \\ & & 1 & 2 & -1 \\ 0 & & & 1 & -1 \\ & & & & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1/2 & 0 & 1/2 & 0 & 0 \\ & -1 & 1 & 0 & -1 \\ & & 1 & -2 & -1 \\ 0 & & & 1 & 1 \\ & & & & 1 \end{bmatrix}\right. \tag{1}$$

Note that the derived unit of charge $\sqrt{\hbar/Z_0}$ is independent of T

$\mathbb{G}$ is the rationalized universal constant of gravitation having units $M^{-1}L^3T^{-2}$ — taking a row vector of the powers of $M^{-1}L^3T^{-2}$ and right multiplying by the matrix of (1) yields a derivation of the appropriate unit

$$\left[\begin{array}{cc|ccc} \scriptstyle Q & \scriptstyle \Theta & \scriptstyle M & \scriptstyle L & \scriptstyle T \\ 0 & 0 & -1 & 3 & -2 \end{array}\right] \overset{(1)}{\mapsto} \left[\begin{array}{cc|ccc} \scriptstyle Z_0 & \scriptstyle k & \scriptstyle \hbar & \scriptstyle c & \scriptstyle T \\ 0 & 0 & -1 & 5 & 2 \end{array}\right] \Rightarrow 1\,M^{-1}L^3T^{-2} = \frac{c^5T^2}{\hbar} \quad \therefore \frac{\mathbb{G}}{1\,M^{-1}L^3T^{-2}} = \left(\frac{T}{\sqrt{\hbar\mathbb{G}/c^5}}\right)^{-2} \tag{2}$$

$T = \sqrt{\hbar\mathbb{G}/c^5}$ for Wholly Derived System of Units at the Planck Scale

$$\begin{array}{rl} Z_0 &= 1\,Q^{-2}ML^2T^{-1} \\ k &= 1\,\Theta^{-1}ML^2T^{-2} \\ \hbar &= 1\,ML^2T^{-1} \\ c &= 1\,LT^{-1} \\ \mathbb{G} &= 1\,M^{-1}L^3T^{-2} \end{array} \Leftrightarrow \left.\begin{array}{rl} 1\,Q &= \sqrt{\hbar/Z_0} \\ 1\,\Theta &= \frac{1}{k}\sqrt{\hbar c^5/\mathbb{G}} \\ 1\,M &= \sqrt{\hbar c/\mathbb{G}} \\ 1\,L &= \sqrt{\hbar\mathbb{G}/c^3} \\ 1\,T &= \sqrt{\hbar\mathbb{G}/c^5} \end{array}\right\}\left\{\left[\begin{array}{cc|ccc} -2 & 0 & 1 & 2 & -1 \\ 0 & -1 & 1 & 2 & -2 \\ \hline & & 1 & 2 & -1 \\ 0 & & 0 & 1 & -1 \\ & & -1 & 3 & -2 \end{array}\right]^{-1} = \left[\begin{array}{cc|ccc} -1/2 & 0 & 1/2 & 0 & 0 \\ 0 & -1 & 1/2 & 5/2 & -1/2 \\ \hline & & 1/2 & 1/2 & -1/2 \\ 0 & & 1/2 & -3/2 & 1/2 \\ & & 1/2 & -5/2 & 1/2 \end{array}\right]\right. \tag{3}$$

Relative Error in Conversions from SI

SI	relative error	via (1)	relative error	via (3)	relative error
Z_0	1.50×10^{-10}	1 Q	7.50×10^{-11}	1 Q	7.50×10^{-11}
k	exactly	1 Θ	exactly*	1 Θ	1.10×10^{-5}
$\hbar$	exactly	1 M	exactly*	1 M	1.10×10^{-5}
c	exactly	1 L	exactly*	1 L	1.10×10^{-5}
$\mathbb{G}$	2.20×10^{-5}	1 T	exactly*	1 T	1.10×10^{-5}

*exactly whenever $\frac{T}{1\,\mathrm{s}}$ is exact

Improbably-Sized Natural Units

$T = 1$ s for Time Derived System of Units (ð – eth) (1 ðt ≡ 1 s) at no sensible scale

$$
\begin{array}{llll}
3.767 \times 10^{2}\,\Omega &= Z_0 = 1\,{}^{\text{ðe·s}}/_{\text{ðq}^2} & & 1\,\text{ðq} = 5.291 \times 10^{-19}\,\text{C} \\
1.381 \times 10^{-23}\,{}^{\text{J}}/_{\text{K}} &= k = 1\,{}^{\text{ðe}}/_{\text{ða}} & \overset{(1)}{\Rightarrow} & 1\,\text{ða} = 7.638 \times 10^{-12}\,\text{K} \\
1.055 \times 10^{-34}\,\text{J·s} &= \hbar = 1\,\text{ðe·s} & & 1\,\text{ðm} = 1.173 \times 10^{-51}\,\text{kg} \\
2.998 \times 10^{8}\,{}^{\text{m}}/_{\text{s}} &= c = 1\,{}^{\text{ðl}}/_{\text{s}} & & 1\,\text{ðl} = 2.998 \times 10^{8}\,\text{m}
\end{array}
$$

$$
\therefore 8.387 \times 10^{-10}\,{}^{\text{N·m}^2}/_{\text{kg}^2} = \mathbb{G} \overset{(2)}{=} 3.652 \times 10^{-86}\,{}^{\text{ðf·ðl}^2}/_{\text{ðm}^2}
$$

$T = \sqrt{\hbar\mathbb{G}/c^5}$ for Wholly Derived System of Units (þ – thorn) at the Planck Scale

$$
\begin{array}{llll}
3.767 \times 10^{2}\,\Omega &= Z_0 = 1\,{}^{\text{þe·þt}}/_{\text{þq}^2} & & 1\,\text{þq} = 5.291 \times 10^{-19}\,\text{C} \\
1.381 \times 10^{-23}\,{}^{\text{J}}/_{\text{K}} &= k = 1\,{}^{\text{þe}}/_{\text{þa}} & & 1\,\text{þa} = 3.997 \times 10^{31}\,\text{K} \\
1.055 \times 10^{-34}\,\text{J·s} &= \hbar = 1\,\text{þe·þt} & \overset{(1\&3)}{\Longrightarrow} & 1\,\text{þm} = 6.140 \times 10^{-9}\,\text{kg} \\
2.998 \times 10^{8}\,{}^{\text{m}}/_{\text{s}} &= c = 1\,{}^{\text{þl}}/_{\text{þt}} & & 1\,\text{þl} = 5.729 \times 10^{-35}\,\text{m} \\
8.387 \times 10^{-10}\,{}^{\text{N·m}^2}/_{\text{kg}^2} &= \mathbb{G} = 1\,{}^{\text{þf·þl}^2}/_{\text{þm}^2} & & 1\,\text{þt} = 1.911 \times 10^{-43}\,\text{s}
\end{array}
$$

T as fractions of 1 s approximating $\sqrt{\hbar\mathbb{G}/c^5}$ for Planck Scale Time Derived Units (þf, þd and þb)

$T = 60^{-24}$ s for Units (þf)

$$
\begin{array}{l}
1\,\text{þfq} = 5.291 \times 10^{-19}\,\text{C} \\
1\,\text{þfa} = 3.619 \times 10^{31}\,\text{K} \\
1\,\text{þfm} = 5.560 \times 10^{-9}\,\text{kg} \\
1\,\text{þfl} = 6.327 \times 10^{-35}\,\text{m} \\
1\,\text{þft} = 2.110 \times 10^{-43}\,\text{s}
\end{array}
$$

$$
\therefore \mathbb{G} \overset{(2)}{=} 0.820\,1\,{}^{\text{þff·þfl}^2}/_{\text{þfm}^2}
$$

$T = 10^{-43}$ s for Units (þd)

$$
\begin{array}{l}
1\,\text{þdq} = 5.291 \times 10^{-19}\,\text{C} \\
1\,\text{þda} = 7.638 \times 10^{31}\,\text{K} \\
1\,\text{þdm} = 1.173 \times 10^{-8}\,\text{kg} \\
1\,\text{þdl} = 2.998 \times 10^{-35}\,\text{m} \\
1\,\text{þdt} = 1.000 \times 10^{-43}\,\text{s}
\end{array}
$$

$$
\therefore \mathbb{G} \overset{(2)}{=} 3.652\,{}^{\text{þdf·þdl}^2}/_{\text{þdm}^2}
$$

$T = 2^{-142}$ s for Units (þb)

$$
\begin{array}{l}
1\,\text{þbq} = 5.291 \times 10^{-19}\,\text{C} \\
1\,\text{þba} = 4.258 \times 10^{31}\,\text{K} \\
1\,\text{þbm} = 6.542 \times 10^{-9}\,\text{kg} \\
1\,\text{þbl} = 5.377 \times 10^{-35}\,\text{m} \\
1\,\text{þbt} = 1.794 \times 10^{-43}\,\text{s}
\end{array}
$$

$$
\therefore \mathbb{G} \overset{(2)}{=} 1.135\,{}^{\text{þbf·þbl}^2}/_{\text{þbm}^2}
$$

Practically-Sized Units As Systems Shifted from the Planck Scale

Shifted (þf) and (þ) Units

$$
\begin{array}{lll}
1\,\text{Fßq} = 60^{9}\,\text{þfq} = 5.332\,\text{mC} \\
1\,{}^{\circ}\text{ßa} = 60^{-18}\,\text{þfa} = 0.356\,4\,\text{K} \\
1\,\text{qßm} = 60^{4}\,\text{þfm} = 72.06\,\text{g} \\
1\,\text{fßl} = 60^{19}\,\text{þfl} = 0.385\,5\,\text{m} \\
1\,\text{Ffq} = 60^{9}\,\text{þq} = 5.332\,\text{mC} \\
1\,{}^{\circ}\text{fa} = 60^{-18}\,\text{þa} = 0.393\,5\,\text{K} \\
1\,\text{qfm} = 60^{4}\,\text{þm} = 79.57\,\text{g} \\
1\,\text{ffl} = 60^{19}\,\text{þl} = 0.349\,1\,\text{m} \\
1\,\text{sft} = 60^{24}\,\text{þt} = 0.905\,6\,\text{s}
\end{array}
$$

Shifted (þd) and (þ) Units

$$
\begin{array}{lll}
1\,\text{Fddq} = 10^{16}\,\text{þdq} = 5.291\,\text{mC} \\
1\,{}^{\circ}\text{dda} = 10^{-32}\,\text{þda} = 0.763\,8\,\text{K} \\
1\,\text{qddm} = 10^{7}\,\text{þdm} = 117.3\,\text{g} \\
1\,\text{fddl} = 10^{34}\,\text{þdl} = 0.299\,8\,\text{m} \\
1\,\text{Fdq} = 10^{16}\,\text{þq} = 5.291\,\text{mC} \\
1\,{}^{\circ}\text{da} = 10^{-32}\,\text{þa} = 0.399\,7\,\text{K} \\
1\,\text{qdm} = 10^{7}\,\text{þm} = 61.40\,\text{g} \\
1\,\text{fdl} = 10^{34}\,\text{þl} = 0.572\,9\,\text{m} \\
1\,\text{sdt} = 10^{43}\,\text{þt} = 1.911\,\text{s}
\end{array}
$$

Shifted (þb) and (þ) Units

$$
\begin{array}{lll}
1\,\text{Fbbq} = 2^{53}\,\text{þbq} = 4.766\,\text{mC} \\
1\,{}^{\circ}\text{bba} = 2^{-106}\,\text{þba} = 0.524\,9\,\text{K} \\
1\,\text{qbbm} = 2^{24}\,\text{þbm} = 109.8\,\text{g} \\
1\,\text{fbbl} = 2^{112}\,\text{þbl} = 0.279\,2\,\text{m} \\
1\,\text{Fbq} = 2^{53}\,\text{þq} = 4.766\,\text{mC} \\
1\,{}^{\circ}\text{ba} = 2^{-106}\,\text{þa} = 0.492\,6\,\text{K} \\
1\,\text{qbm} = 2^{24}\,\text{þm} = 103.0\,\text{g} \\
1\,\text{fbl} = 2^{112}\,\text{þl} = 0.297\,5\,\text{m} \\
1\,\text{sbt} = 2^{142}\,\text{þt} = 1.065\,5\,\text{s}
\end{array}
$$

The fundamental constants of nature Z_0, k, $\hbar$, c and, for the wholly derived units, $\mathbb{G}$ will be represented by whole powers of the respective base for these sexagenary, denary and binary shifted units.

		f	d	b	
F•	=	60^{9}	10^{16}	2^{53}	þ
°•	=	60^{-18}	10^{-32}	2^{-106}	þ
q•	=	60^{4}	10^{7}	2^{24}	þ
f•	=	60^{19}	10^{34}	2^{112}	þ
s•	=	60^{24}	10^{43}	2^{142}	þ

$\Rightarrow$

		ß, f	dd, d	bb, b	
Z_0	=, =	1	1	1	erg•e·s•t/F•q²
k	=, =	60^{-12}	10^{-21}	2^{-70}	erg•e/°•a
$\hbar$	=, =	60^{-18}	10^{-32}	2^{-106}	erg•e·s•t
c	=, =	60^{5}	10^{9}	2^{30}	f•l/s•t
$\mathbb{G}$	≈, =	60^{-5}	10^{-9}	2^{-28}	dyn•f·f•l²/q•m²

Practically Sized Constants Leading to Practically Sized Units

The constant of nature Z_0 derives a practically sized unit. Dividing other constants k, $\hbar = {}^{h}/_{2\pi}$, c and, for a wholly derived system, $\mathbb{G} = 4\pi G$ by powers of 60, 10 or 2 before calculating the wholly derived or the $T = 1\,\mathrm{s}$ derived base units yields sexagenary, denary and binary units at a convenient scale. For example, dividing c by 60^5 before the derivation yields units T and L such that $c = 1{\times}60^5\,\mathrm{LT^{-1}}$. Weirder powers of 20, 12 or 3 would yield vicenary, duodenary and ternary systems.

foundational constants		scale factors					
Z_0	$\div$	1	1	1	1	1	1
k	$\div$	60^{-12}	20^{-16}	12^{-20}	10^{-21}	3^{-44}	2^{-70}
$\hbar$	$\div$	60^{-18}	20^{-24}	12^{-30}	10^{-32}	3^{-67}	2^{-106}
c	$\div$	60^{5}	20^{7}	12^{8}	10^{9}	3^{19}	2^{30}
$\mathbb{G}$	$\div$	60^{-5}	20^{-7}	12^{-9}	10^{-9}	3^{-17}	2^{-28}

$\Longrightarrow$ e.g. in wholly derived binary units

$$\begin{aligned} Z_0 &= 1\,\mathrm{Q^{-2}ML^2T^{-1}} \\ k &= 1{\times}2^{-70}\,\mathrm{\Theta^{-1}ML^2T^{-2}} \\ \hbar &= 1{\times}2^{-106}\,\mathrm{ML^2T^{-1}} \\ c &= 1{\times}2^{30}\,\mathrm{LT^{-1}} \\ \mathbb{G} &= 1{\times}2^{-28}\,\mathrm{M^{-1}L^3T^{-2}} \end{aligned}$$

e.g. Binary Systems of Units (bb and b)

time	length	mass	force	work	temp.	charge
sbbt	fbbl	qbbm	dynbbf	ergbbe	°bba	Fbbq
sbt	fbl	qbm	dynbf	ergbe	°ba	Fbq

$$\begin{array}{llll} Z_0 & = 0.3767\,\mathrm{k\Omega} & = 1\,{}^{\mathrm{ergbbe\cdot sbbt}}/_{\mathrm{Fbbq^2}} & \\ k \div 2^{-70} & = 16.30\,{}^{\mathrm{mJ}}/_{\mathrm{K}} & = 1\,{}^{\mathrm{ergbbe}}/_{\mathrm{°bba}} & \\ \hbar \div 2^{-106} & = 8.5557\,\mathrm{mJ{\cdot}s} & = 1\,\mathrm{ergbbe{\cdot}sbbt} & \overset{(1)}{\Rightarrow} \\ c \div 2^{30} & = 0.2792\,{}^{\mathrm{m}}/_{\mathrm{s}} & = 1\,{}^{\mathrm{fbbl}}/_{\mathrm{sbbt}} & \\ T & = 1\,\mathrm{s} & = 1\,\mathrm{sbbt} & \end{array} \quad \begin{array}{ll} 1\,\mathrm{Fbbq} & = 4.766\,\mathrm{mC} \\ 1\,\mathrm{°bba} & = 0.5249\,\mathrm{K} \\ 1\,\mathrm{qbbm} & = 109.8\,\mathrm{g} \\ 1\,\mathrm{fbbl} & = 0.2792\,\mathrm{m} \\ 1\,\mathrm{sbbt} & = 1\,\mathrm{s} \end{array}$$

$$1.135{\times}2^{-28}\,{}^{\mathrm{dynbbf\cdot fbbl^2}}/_{\mathrm{qbbm^2}} \overset{(2)}{=} \mathbb{G} = 1{\times}2^{-28}\,{}^{\mathrm{dynbf\cdot fbl^2}}/_{\mathrm{qbm^2}}$$

$$\begin{array}{llll} Z_0 & = 0.3767\,\mathrm{k\Omega} & = 1\,{}^{\mathrm{ergbe\cdot sbt}}/_{\mathrm{Fbq^2}} & \\ k \div 2^{-70} & = 16.30\,{}^{\mathrm{mJ}}/_{\mathrm{K}} & = 1\,{}^{\mathrm{ergbe}}/_{\mathrm{°ba}} & \\ \hbar \div 2^{-106} & = 8.5557\,\mathrm{mJ{\cdot}s} & = 1\,\mathrm{ergbe{\cdot}sbt} & \overset{(3)}{\Rightarrow} \\ c \div 2^{30} & = 0.2792\,{}^{\mathrm{m}}/_{\mathrm{s}} & = 1\,{}^{\mathrm{fbl}}/_{\mathrm{sbt}} & \\ \mathbb{G} \div 2^{-28} & = 0.2251\,{}^{\mathrm{N\cdot m^2}}/_{\mathrm{kg^2}} & = 1\,{}^{\mathrm{dynbf\cdot fbl^2}}/_{\mathrm{qbm^2}} & \end{array} \quad \begin{array}{ll} 1\,\mathrm{Fbq} & = 4.766\,\mathrm{mC} \\ 1\,\mathrm{°ba} & = 0.4926\,\mathrm{K} \\ 1\,\mathrm{qbm} & = 103.0\,\mathrm{g} \\ 1\,\mathrm{fbl} & = 0.2975\,\mathrm{m} \\ 1\,\mathrm{sbt} & = 1.0655\,\mathrm{s} \end{array}$$

$$\begin{array}{llll} 1\,{}^{\mathrm{ergbbe\cdot sbbt}}/_{\mathrm{Fbbq^2}} & = Z_0 & = 1\,{}^{\mathrm{ðe\cdot s}}/_{\mathrm{ðq^2}} & \\ 2^{-70}\,{}^{\mathrm{ergbbe}}/_{\mathrm{°bba}} & = k & = 1\,{}^{\mathrm{ðe}}/_{\mathrm{ða}} & \\ 2^{-106}\,\mathrm{ergbbe{\cdot}sbbt} & = \hbar & = 1\,\mathrm{ðe{\cdot}s} & \overset{(1)}{\Rightarrow} \\ 2^{30}\,{}^{\mathrm{fbbl}}/_{\mathrm{sbbt}} & = c & = 1\,{}^{\mathrm{ðl}}/_{\mathrm{s}} & \\ 1\,\mathrm{sbbt} & = T & = 1\,\mathrm{ðt} & \end{array} \begin{array}{ll} 1\,\mathrm{ðq} & = 2^{-53}\,\mathrm{Fbbq} \\ 1\,\mathrm{ða} & = 2^{-36}\,\mathrm{°bba} \\ 1\,\mathrm{ðm} & = 2^{-166}\,\mathrm{qbbm} \\ 1\,\mathrm{ðl} & = 2^{30}\,\mathrm{fbbl} \\ 1\,\mathrm{ðt} & = 1\,\mathrm{sbbt} \end{array} \Rightarrow \begin{array}{lll} 1\,\mathrm{Fbbq} & = 2^{53}\,\mathrm{ðq} & = 4.766\,\mathrm{mC} \\ 1\,\mathrm{°bba} & = 2^{36}\,\mathrm{ða} & = 0.5249\,\mathrm{K} \\ 1\,\mathrm{qbbm} & = 2^{166}\,\mathrm{ðm} & = 109.8\,\mathrm{g} \\ 1\,\mathrm{fbbl} & = 2^{-20}\,\mathrm{ðl} & = 0.2792\,\mathrm{m} \\ 1\,\mathrm{sbbt} & = 1\,\mathrm{ðqt} & = 1\,\mathrm{s} \end{array}$$

$$\begin{array}{llll} 1\,{}^{\mathrm{ergbe\cdot sbt}}/_{\mathrm{Fbq^2}} & = Z_0 & = 1\,{}^{\mathrm{þe\cdot þt}}/_{\mathrm{þq^2}} & \\ 2^{-70}\,{}^{\mathrm{ergbe}}/_{\mathrm{°ba}} & = k & = 1\,{}^{\mathrm{þe}}/_{\mathrm{þa}} & \\ 2^{-106}\,\mathrm{ergbe{\cdot}sbt} & = \hbar & = 1\,\mathrm{þe{\cdot}þt} & \overset{(3)}{\Rightarrow} \\ 2^{30}\,{}^{\mathrm{fbl}}/_{\mathrm{sbt}} & = c & = 1\,{}^{\mathrm{þl}}/_{\mathrm{þt}} & \\ 2^{-28}\,{}^{\mathrm{dynbf\cdot fbl^2}}/_{\mathrm{qbm^2}} & = \mathbb{G} & = 1\,{}^{\mathrm{þf\cdot þl^2}}/_{\mathrm{þm^2}} & \end{array} \begin{array}{ll} 1\,\mathrm{þq} & = 2^{-53}\,\mathrm{Fbq} \\ 1\,\mathrm{þa} & = 2^{106}\,\mathrm{°ba} \\ 1\,\mathrm{þm} & = 2^{-24}\,\mathrm{qbm} \\ 1\,\mathrm{þl} & = 2^{-112}\,\mathrm{fbl} \\ 1\,\mathrm{þt} & = 2^{-142}\,\mathrm{sbt} \end{array} \Rightarrow \begin{array}{lll} 1\,\mathrm{Fbq} & = 2^{53}\,\mathrm{þq} & = 4.766\,\mathrm{mC} \\ 1\,\mathrm{°ba} & = 2^{-106}\,\mathrm{þa} & = 0.4926\,\mathrm{K} \\ 1\,\mathrm{qbm} & = 2^{24}\,\mathrm{þm} & = 103.0\,\mathrm{g} \\ 1\,\mathrm{fbl} & = 2^{112}\,\mathrm{þl} & = 0.2975\,\mathrm{m} \\ 1\,\mathrm{sbt} & = 2^{142}\,\mathrm{þt} & = 1.0655\,\mathrm{s} \end{array}$$

In the first derivation for practically-sized binary units (bb and b) we use the scaled constants to derive the base units directly through the (1) and (3) transformations. In the second derivation we only use the binary scaling factor to determine the desired value of these fundamental constants in terms of the to-be-derived units. We can then use our (1) and (3) transformation to back derive the improbably sized ð and þ in terms our to-be-derived bb and b units. After flipping this relationship to express the new binary units in terms of the ð and þ (i.e. shifted from the ð and þ), we can then express the binary units in SI. This presentation links our two methods of deriving practically sized units.

Sexagenary Divided Units (ß≡ſſ and ſ — long and sharp s)

time	length	mass	force	work	temp.	charge
s	fßl	qßm	dynßf	ergße	°ßa	Fßq
sſt	fſl	qſm	dynſf	ergſe	°ſa	Fſq

$$\begin{array}{llll}
Z_0 & = 0.376\,7\,\mathrm{k\Omega} & = 1\,{}^{\text{ergße·s}}/_{\text{Fßq}^2} & 1\,\text{Fßq} = 5.332\,\mathrm{mC} \\
k \div 60^{-12} & = 30.05\,{}^{\mathrm{mJ}}/_{\mathrm{K}} & = 1\,{}^{\text{ergße}}/_{\text{°ßa}} \quad \overset{(1)}{\Rightarrow} & 1\,\text{°ßa} = 0.356\,4\,\mathrm{K} \\
\hbar \div 60^{-18} & = 10.71\,\mathrm{mJ{\cdot}s} & = 1\,\text{ergße·s} & 1\,\text{qßm} = 72.06\,\mathrm{g} \\
c \div 60^{5} & = 0.385\,5\,{}^{\mathrm{m}}/_{\mathrm{s}} & = 1\,{}^{\text{fßl}}/_{\text{s}} & 1\,\text{fßl} = 0.385\,5\,\mathrm{m}
\end{array}$$

$$0.820\,1\times 60^{-5}\,{}^{\text{dynßf·fßl}^2}/_{\text{qßm}^2} \overset{(2)}{=} \mathbb{G} = 1\times 60^{-5}\,{}^{\text{dynſf·fſl}^2}/_{\text{qſm}^2}$$

$$\begin{array}{llll}
Z_0 & = 0.376\,7\,\mathrm{k\Omega} & = 1\,{}^{\text{ergſe·sſt}}/_{\text{Fſq}^2} & 1\,\text{Fſq} = 5.332\,\mathrm{mC} \\
k \div 60^{-12} & = 30.05\,{}^{\mathrm{mJ}}/_{\mathrm{K}} & = 1\,{}^{\text{ergſe}}/_{\text{°ſa}} & 1\,\text{°ſa} = 0.393\,5\,\mathrm{K} \\
\hbar \div 60^{-18} & = 10.71\,\mathrm{mJ{\cdot}s} & = 1\,\text{ergſe·sſt} \quad \overset{(3)}{\Rightarrow} & 1\,\text{qſm} = 79.57\,\mathrm{g} \\
c \div 60^{5} & = 0.385\,5\,{}^{\mathrm{m}}/_{\mathrm{s}} & = 1\,{}^{\text{fſl}}/_{\text{sſt}} & 1\,\text{fſl} = 0.349\,1\,\mathrm{m} \\
\mathbb{G} \div 60^{-5} & = 0.652\,2\,{}^{\mathrm{N{\cdot}m^2}}/_{\mathrm{kg^2}} & = 1\,{}^{\text{dynſf·fſl}^2}/_{\text{qſm}^2} & 1\,\text{sſt} = 0.905\,6\,\mathrm{s}
\end{array}$$

Denary Divided Units (dd and d)

time	length	mass	force	work	temp.	charge
s	fddl	qddm	dynddf	ergdde	°dda	Fddq
sdt	fdl	qdm	dyndf	ergde	°da	Fdq

$$\begin{array}{llll}
Z_0 & = 0.376\,7\,\mathrm{k\Omega} & = 1\,{}^{\text{ergdde·s}}/_{\text{Fddq}^2} & 1\,\text{Fddq} = 5.291\,\mathrm{mC} \\
k \div 10^{-21} & = 13.81\,{}^{\mathrm{mJ}}/_{\mathrm{K}} & = 1\,{}^{\text{ergdde}}/_{\text{°dda}} \quad \overset{(1)}{\Rightarrow} & 1\,\text{°dda} = 0.763\,8\,\mathrm{K} \\
\hbar \div 10^{-32} & = 10.55\,\mathrm{mJ{\cdot}s} & = 1\,\text{ergdde·s} & 1\,\text{qddm} = 117.3\,\mathrm{g} \\
c \div 10^{9} & = 0.299\,8\,{}^{\mathrm{m}}/_{\mathrm{s}} & = 1\,{}^{\text{fddl}}/_{\text{s}} & 1\,\text{fddl} = 0.299\,8\,\mathrm{m}
\end{array}$$

$$3.652\times 10^{-9}\,{}^{\text{dynddf·fddl}^2}/_{\text{qddm}^2} \overset{(2)}{=} \mathbb{G} = 1\times 10^{-9}\,{}^{\text{dyndf·fdl}^2}/_{\text{qdm}^2}$$

$$\begin{array}{llll}
Z_0 & = 0.376\,7\,\mathrm{k\Omega} & = 1\,{}^{\text{ergde·sdt}}/_{\text{Fdq}^2} & 1\,\text{Fdq} = 5.291\,\mathrm{mC} \\
k \div 10^{-21} & = 13.81\,{}^{\mathrm{mJ}}/_{\mathrm{K}} & = 1\,{}^{\text{ergde}}/_{\text{°da}} & 1\,\text{°da} = 0.399\,7\,\mathrm{K} \\
\hbar \div 10^{-32} & = 10.55\,\mathrm{mJ{\cdot}s} & = 1\,\text{ergde·sdt} \quad \overset{(3)}{\Rightarrow} & 1\,\text{qdm} = 61.40\,\mathrm{g} \\
c \div 10^{9} & = 0.299\,8\,{}^{\mathrm{m}}/_{\mathrm{s}} & = 1\,{}^{\text{fdl}}/_{\text{sdt}} & 1\,\text{fdl} = 0.572\,9\,\mathrm{m} \\
\mathbb{G} \div 10^{-9} & = 0.838\,7\,{}^{\mathrm{N{\cdot}m^2}}/_{\mathrm{kg^2}} & = 1\,{}^{\text{dyndf·fdl}^2}/_{\text{qdm}^2} & 1\,\text{sdt} = 1.911\,\mathrm{s}
\end{array}$$

Binary Divided Units (bb and b)

time	length	mass	force	work	temp.	charge
s	fbbl	qbbm	dynbbf	ergbbe	°bba	Fbbq
sbt	fbl	qbm	dynbf	ergbe	°ba	Fbq

$$\begin{array}{llll}
Z_0 & = 0.376\,7\,\mathrm{k\Omega} & = 1\,{}^{\text{ergbbe·s}}/_{\text{Fbbq}^2} & 1\,\text{Fbbq} = 4.766\,\mathrm{mC} \\
k \div 2^{-70} & = 16.30\,{}^{\mathrm{mJ}}/_{\mathrm{K}} & = 1\,{}^{\text{ergbbe}}/_{\text{°bba}} \quad \overset{(1)}{\Rightarrow} & 1\,\text{°bba} = 0.524\,9\,\mathrm{K} \\
\hbar \div 2^{-106} & = 8.555\,7\,\mathrm{mJ{\cdot}s} & = 1\,\text{ergbbe·s} & 1\,\text{qbbm} = 109.8\,\mathrm{g} \\
c \div 2^{30} & = 0.279\,2\,{}^{\mathrm{m}}/_{\mathrm{s}} & = 1\,{}^{\text{fbbl}}/_{\text{s}} & 1\,\text{fbbl} = 0.279\,2\,\mathrm{m}
\end{array}$$

$$1.135\times 2^{-28}\,{}^{\text{dynbbf·fbbl}^2}/_{\text{qbbm}^2} \overset{(2)}{=} \mathbb{G} = 1\times 2^{-28}\,{}^{\text{dynbf·fbl}^2}/_{\text{qbm}^2}$$

$$\begin{array}{llll}
Z_0 & = 0.376\,7\,\mathrm{k\Omega} & = 1\,{}^{\text{ergbe·sbt}}/_{\text{Fbq}^2} & 1\,\text{Fbq} = 4.766\,\mathrm{mC} \\
k \div 2^{-70} & = 16.30\,{}^{\mathrm{mJ}}/_{\mathrm{K}} & = 1\,{}^{\text{ergbe}}/_{\text{°ba}} & 1\,\text{°ba} = 0.492\,6\,\mathrm{K} \\
\hbar \div 2^{-106} & = 8.555\,7\,\mathrm{mJ{\cdot}s} & = 1\,\text{ergbe·sbt} \quad \overset{(3)}{\Rightarrow} & 1\,\text{qbm} = 103.0\,\mathrm{g} \\
c \div 2^{30} & = 0.279\,2\,{}^{\mathrm{m}}/_{\mathrm{s}} & = 1\,{}^{\text{fbl}}/_{\text{sbt}} & 1\,\text{fbl} = 0.297\,5\,\mathrm{m} \\
\mathbb{G} \div 2^{-28} & = 0.225\,1\,{}^{\mathrm{N{\cdot}m^2}}/_{\mathrm{kg^2}} & = 1\,{}^{\text{dynbf·fbl}^2}/_{\text{qbm}^2} & 1\,\text{sbt} = 1.065\,5\,\mathrm{s}
\end{array}$$

Vicenary Divided Units (ϕϕ and ϕ)

time	length	mass	force	work	temp.	charge
s	fϕϕl	dynϕϕf	ergϕϕe	qϕϕm	°ϕϕa	Fϕϕq
sϕt	fϕl	dynϕf	ergϕe	qϕm	°ϕa	Fϕq

$$\begin{aligned}
Z_0 &= 0.3767\,\mathrm{k\Omega} &&= 1\,{}^{\text{ergϕϕe·s}}/_{\text{Fϕϕq}^2} & & 1\,\text{Fϕϕq} &&= 2.167\,\mathrm{mC}\\
k \div 20^{-16} &= 9.048\,{}^{\mathrm{mJ}}/_{\mathrm{K}} &&= 1\,{}^{\text{ergϕϕe}}/_{\text{°ϕϕa}} & \overset{(1)}{\Rightarrow} & 1\,\text{°ϕϕa} &&= 0.1956\,\mathrm{K}\\
\hbar \div 20^{-24} &= 1.769\,\mathrm{mJ\cdot s} &&= 1\,\text{ergϕϕe·s} & & 1\,\text{qϕϕm} &&= 32.25\,\mathrm{g}\\
c \div 20^{7} &= 0.2342\,{}^{\mathrm{m}}/_{\mathrm{s}} &&= 1\,{}^{\text{fϕϕl}}/_{\text{s}} & & 1\,\text{fϕϕl} &&= 0.2342\,\mathrm{m}
\end{aligned}$$

$$2.695\times 20^{-7}\,{}^{\text{dynϕϕf·fϕϕl}^2}/_{\text{qϕϕm}^2} \overset{(2)}{=} \mathbb{G} = 1\times 20^{-7}\,{}^{\text{dynϕf·fϕl}^2}/_{\text{qϕm}^2}$$

$$\begin{aligned}
Z_0 &= 0.3767\,\mathrm{k\Omega} &&= 1\,{}^{\text{ergϕe·sϕt}}/_{\text{Fϕq}^2} & & 1\,\text{Fϕq} &&= 2.167\,\mathrm{mC}\\
k \div 20^{-16} &= 9.048\,{}^{\mathrm{mJ}}/_{\mathrm{K}} &&= 1\,{}^{\text{ergϕe}}/_{\text{°ϕa}} & & 1\,\text{°ϕa} &&= 0.1191\,\mathrm{K}\\
\hbar \div 20^{-24} &= 1.769\,\mathrm{mJ\cdot s} &&= 1\,\text{ergϕe·sϕt} & \overset{(3)}{\Rightarrow} & 1\,\text{qϕm} &&= 19.65\,\mathrm{g}\\
c \div 20^{7} &= 0.2342\,{}^{\mathrm{m}}/_{\mathrm{s}} &&= 1\,{}^{\text{fϕl}}/_{\text{sϕt}} & & 1\,\text{fϕl} &&= 0.3845\,\mathrm{m}\\
\mathbb{G} \div 20^{-7} &= 1.073\,{}^{\mathrm{N\cdot m^2}}/_{\mathrm{kg^2}} &&= 1\,{}^{\text{dynϕf·fϕl}^2}/_{\text{qϕm}^2} & & 1\,\text{sϕt} &&= 1.642\,\mathrm{s}
\end{aligned}$$

Duodenary Divided Units (ȸȸ and ȸ)

time	length	mass	force	work	temp.	charge
s	fȸȸl	qȸȸm	dynȸȸf	ergȸȸe	°ȸȸa	Fȸȸq
sȸt	fȸl	qȸm	dynȸf	ergȸe	°ȸa	Fȸq

$$\begin{aligned}
Z_0 &= 0.3767\,\mathrm{k\Omega} &&= 1\,{}^{\text{ergȸȸe·s}}/_{\text{Fȸȸq}^2} & & 1\,\text{Fȸȸq} &&= 8.152\,\mathrm{mC}\\
k \div 12^{-20} &= 52.93\,{}^{\mathrm{mJ}}/_{\mathrm{K}} &&= 1\,{}^{\text{ergȸȸe}}/_{\text{°ȸȸa}} & \overset{(1)}{\Rightarrow} & 1\,\text{°ȸȸa} &&= 0.4729\,\mathrm{K}\\
\hbar \div 12^{-30} &= 25.03\,\mathrm{mJ\cdot s} &&= 1\,\text{ergȸȸe·s} & & 1\,\text{qȸȸm} &&= 51.50\,\mathrm{g}\\
c \div 12^{8} &= 0.6972\,{}^{\mathrm{m}}/_{\mathrm{s}} &&= 1\,{}^{\text{fȸȸl}}/_{\text{s}} & & 1\,\text{fȸȸl} &&= 0.6972\,\mathrm{m}
\end{aligned}$$

$$0.6575\times 12^{-9}\,{}^{\text{dynȸȸf·fȸȸl}^2}/_{\text{qȸȸm}^2} \overset{(2)}{=} \mathbb{G} = 1\times 12^{-9}\,{}^{\text{dynȸf·fȸl}^2}/_{\text{qȸm}^2}$$

$$\begin{aligned}
Z_0 &= 0.3767\,\mathrm{k\Omega} &&= 1\,{}^{\text{ergȸe·sȸt}}/_{\text{Fȸq}^2} & & 1\,\text{Fȸq} &&= 8.152\,\mathrm{mC}\\
k \div 12^{-20} &= 52.93\,{}^{\mathrm{mJ}}/_{\mathrm{K}} &&= 1\,{}^{\text{ergȸe}}/_{\text{°ȸa}} & & 1\,\text{°ȸa} &&= 0.5833\,\mathrm{K}\\
\hbar \div 12^{-30} &= 25.03\,\mathrm{mJ\cdot s} &&= 1\,\text{ergȸe·sȸt} & \overset{(3)}{\Rightarrow} & 1\,\text{qȸm} &&= 63.51\,\mathrm{g}\\
c \div 12^{8} &= 0.6972\,{}^{\mathrm{m}}/_{\mathrm{s}} &&= 1\,{}^{\text{fȸl}}/_{\text{sȸt}} & & 1\,\text{fȸl} &&= 0.5654\,\mathrm{m}\\
\mathbb{G} \div 12^{-9} &= 4.327\,{}^{\mathrm{N\cdot m^2}}/_{\mathrm{kg^2}} &&= 1\,{}^{\text{dynȸf·fȸl}^2}/_{\text{qȸm}^2} & & 1\,\text{sȸt} &&= 0.8109\,\mathrm{s}
\end{aligned}$$

Ternary Divided Units (θθ and θ)

time	length	mass	force	work	temp.	charge
s	fθθl	qθθm	dynθθf	ergθθe	°θθa	Fθθq
sθt	fθl	qθm	dynθf	ergθe	°θa	Fθq

$$\begin{aligned}
Z_0 &= 0.3767\,\mathrm{k\Omega} &&= 1\,{}^{\text{ergθθe·s}}/_{\text{Fθθq}^2} & & 1\,\text{Fθθq} &&= 5.094\,\mathrm{mC}\\
k \div 3^{-40} &= 13.60\,{}^{\mathrm{mJ}}/_{\mathrm{K}} &&= 1\,{}^{\text{ergθθe}}/_{\text{°θθa}} & \overset{(1)}{\Rightarrow} & 1\,\text{°θθa} &&= 0.7191\,\mathrm{K}\\
\hbar \div 3^{-67} &= 9.777\,\mathrm{mJ\cdot s} &&= 1\,\text{ergθθe·s} & & 1\,\text{qθθm} &&= 146\,9\,\mathrm{g}\\
c \div 3^{19} &= 0.2579\,{}^{\mathrm{m}}/_{\mathrm{s}} &&= 1\,{}^{\text{fθθl}}/_{\text{s}} & & 1\,\text{fθθl} &&= 0.2579\,\mathrm{m}
\end{aligned}$$

$$0.9275\times 3^{-17}\,{}^{\text{dynθθf·fθθl}^2}/_{\text{qθθm}^2} \overset{(2)}{=} \mathbb{G} = 1\times 3^{-17}\,{}^{\text{dynθf·fθl}^2}/_{\text{qθm}^2}$$

$$\begin{aligned}
Z_0 &= 0.3767\,\mathrm{k\Omega} &&= 1\,{}^{\text{ergθe·sθt}}/_{\text{Fθq}^2} & & 1\,\text{Fθq} &&= 5.094\,\mathrm{mC}\\
k \div 3^{-40} &= 13.60\,{}^{\mathrm{mJ}}/_{\mathrm{K}} &&= 1\,{}^{\text{ergθe}}/_{\text{°θa}} & & 1\,\text{°θa} &&= 0.7467\,\mathrm{K}\\
\hbar \div 3^{-67} &= 9.777\,\mathrm{mJ\cdot s} &&= 1\,\text{ergθe·sθt} & \overset{(3)}{\Rightarrow} & 1\,\text{qθm} &&= 152\,6\,\mathrm{g}\\
c \div 3^{19} &= 0.2579\,{}^{\mathrm{m}}/_{\mathrm{s}} &&= 1\,{}^{\text{fθl}}/_{\text{sθt}} & & 1\,\text{fθl} &&= 0.2484\,\mathrm{m}\\
\mathbb{G} \div 3^{-17} &= 0.1083\,{}^{\mathrm{N\cdot m^2}}/_{\mathrm{kg^2}} &&= 1\,{}^{\text{dynθf·fθl}^2}/_{\text{qθm}^2} & & 1\,\text{sθt} &&= 0.9630\,\mathrm{s}
\end{aligned}$$

Sexagenary Units (ß and ſ) through the ð and þ and thereby Shifted

$$\begin{array}{llll}
1\ \text{ergße}\cdot\text{s}/\text{Fßq}^2 = Z_0 = 1\ \text{ðe}\cdot\text{s}/\text{ðq}^2 & & 1\ \text{ðq} = 60^{-9}\ \text{Fßq} & & 1\ \text{Fßq} = 60^{9}\ \text{ðq} = 5.332\ \text{mC} \\
60^{-12}\ \text{ergße}/{}^\circ\text{ßa} = k = 1\ \text{ðe}/\text{ða} & \overset{(1)}{\Rightarrow} & 1\ \text{ða} = 60^{-6}\ {}^\circ\text{ßa} & \Rightarrow & 1\ {}^\circ\text{ßa} = 60^{6}\ \text{ða} = 0.3564\ \text{K} \\
60^{-18}\ \text{ergße}\cdot\text{s} = \hbar = 1\ \text{ðe}\cdot\text{s} & & 1\ \text{ðm} = 60^{-28}\ \text{qßm} & & 1\ \text{qßm} = 60^{28}\ \text{ðm} = 72.06\ \text{g} \\
60^{5}\ \text{fßl}/\text{s} = c = 1\ \text{ðl}/\text{s} & & 1\ \text{ðl} = 60^{5}\ \text{fßl} & & 1\ \text{fßl} = 60^{-5}\ \text{ðl} = 0.3855\ \text{m}
\end{array}$$

$$\begin{array}{lllll}
1\ \text{ergſe}\cdot\text{sſt}/\text{Fſq}^2 = Z_0 = 1\ \text{þe}\cdot\text{þt}/\text{þq}^2 & & 1\ \text{þq} = 60^{-9}\ \text{Fſq} & & 1\ \text{Fſq} = 60^{9}\ \text{þq} = 5.332\ \text{mC} \\
60^{-12}\ \text{ergſe}/{}^\circ\text{ſa} = k = 1\ \text{þe}/\text{þa} & & 1\ \text{þa} = 60^{18}\ {}^\circ\text{ſa} & & 1\ {}^\circ\text{ſa} = 60^{-18}\ \text{þa} = 0.3935\ \text{K} \\
60^{-18}\ \text{ergſe}\cdot\text{sſt} = \hbar = 1\ \text{þe}\cdot\text{þt} & \overset{(3)}{\Rightarrow} & 1\ \text{þm} = 60^{-4}\ \text{qſm} & \Rightarrow & 1\ \text{qſm} = 60^{4}\ \text{þm} = 79.57\ \text{g} \\
60^{5}\ \text{fſl}/\text{sſt} = c = 1\ \text{þl}/\text{þt} & & 1\ \text{þl} = 60^{-19}\ \text{fſl} & & 1\ \text{fſl} = 60^{19}\ \text{þl} = 0.3491\ \text{m} \\
60^{-5}\ \text{dynſf}\cdot\text{fſl}^2/\text{qſm}^2 = \mathbb{G} = 1\ \text{þf}\cdot\text{þl}^2/\text{þm}^2 & & 1\ \text{þt} = 60^{-24}\ \text{sſt} & & 1\ \text{sſt} = 60^{24}\ \text{þt} = 0.9056\ \text{s}
\end{array}$$

$$\therefore \mathbb{G} = 0.82004 \times 60^{-5}\ \text{dynßf}\cdot\text{fßl}^2/\text{qßm}^2 \qquad \because 1\ \text{ðq} \equiv 1\ \text{þq} \quad \therefore 1\ \text{Fßq} \equiv 1\ \text{Fſq}$$

Denary Units (dd and d) through the ð and þ and thereby Shifted

$$\begin{array}{lllll}
1\ \text{ergdde}\cdot\text{s}/\text{Fddq}^2 = Z_0 = 1\ \text{ðe}\cdot\text{s}/\text{ðq}^2 & & 1\ \text{ðq} = 10^{-16}\ \text{Fddq} & & 1\ \text{Fddq} = 10^{16}\ \text{ðq} = 5.291\ \text{mC} \\
10^{-21}\ \text{ergdde}/{}^\circ\text{dda} = k = 1\ \text{ðe}/\text{ða} & \overset{(1)}{\Rightarrow} & 1\ \text{ða} = 10^{-11}\ {}^\circ\text{dda} & \Rightarrow & 1\ {}^\circ\text{dda} = 10^{11}\ \text{ða} = 0.7638\ \text{K} \\
10^{-32}\ \text{ergdde}\cdot\text{s} = \hbar = 1\ \text{ðe}\cdot\text{s} & & 1\ \text{ðm} = 10^{-50}\ \text{qddm} & & 1\ \text{qddm} = 10^{50}\ \text{ðm} = 117.3\ \text{g} \\
10^{9}\ \text{fddl}/\text{s} = c = 1\ \text{ðl}/\text{s} & & 1\ \text{ðl} = 10^{9}\ \text{fddl} & & 1\ \text{fddl} = 10^{-9}\ \text{ðl} = 0.2998\ \text{m}
\end{array}$$

$$\begin{array}{lllll}
1\ \text{ergde}\cdot\text{sdt}/\text{Fdq}^2 = Z_0 = 1\ \text{þe}\cdot\text{þt}/\text{þq}^2 & & 1\ \text{þq} = 10^{-16}\ \text{Fdq} & & 1\ \text{Fdq} = 10^{16}\ \text{þq} = 5.291\ \text{mC} \\
10^{-21}\ \text{ergde}/{}^\circ\text{da} = k = 1\ \text{þe}/\text{þa} & & 1\ \text{þa} = 10^{32}\ {}^\circ\text{da} & & 1\ {}^\circ\text{da} = 10^{-32}\ \text{þa} = 0.3997\ \text{K} \\
10^{-32}\ \text{ergde}\cdot\text{sdt} = \hbar = 1\ \text{þe}\cdot\text{þt} & \overset{(3)}{\Rightarrow} & 1\ \text{þm} = 10^{-7}\ \text{qdm} & \Rightarrow & 1\ \text{qdm} = 10^{7}\ \text{þm} = 61.40\ \text{g} \\
10^{9}\ \text{fdl}/\text{sdt} = c = 1\ \text{þl}/\text{þt} & & 1\ \text{þl} = 10^{-34}\ \text{fdl} & & 1\ \text{fdl} = 10^{34}\ \text{þl} = 0.5729\ \text{m} \\
10^{-9}\ \text{dyndf}\cdot\text{fdl}^2/\text{qdm}^2 = \mathbb{G} = 1\ \text{þf}\cdot\text{þl}^2/\text{þm}^2 & & 1\ \text{þt} = 10^{-43}\ \text{sdt} & & 1\ \text{sdt} = 10^{43}\ \text{þt} = 1.911\ \text{s}
\end{array}$$

$$\therefore \mathbb{G} = 3.652 \times 10^{-9}\ \text{dynddf}\cdot\text{fddl}^2/\text{qddm}^2 \qquad \because 1\ \text{ðq} \equiv 1\ \text{þq} \quad \therefore 1\ \text{Fddq} \equiv 1\ \text{Fdq}$$

Binary Units (bb and b) through the ð and þ and thereby Shifted

$$\begin{array}{lllll}
1\ \text{ergbbe}\cdot\text{s}/\text{Fbbq}^2 = Z_0 = 1\ \text{ðe}\cdot\text{s}/\text{ðq}^2 & & 1\ \text{ðq} = 2^{-53}\ \text{Fbbq} & & 1\ \text{Fbbq} = 2^{53}\ \text{ðq} = 4.766\ \text{mC} \\
2^{-70}\ \text{ergbbe}/{}^\circ\text{bba} = k = 1\ \text{ðe}/\text{ða} & \overset{(1)}{\Rightarrow} & 1\ \text{ða} = 2^{-36}\ {}^\circ\text{bba} & \Rightarrow & 1\ {}^\circ\text{bba} = 2^{36}\ \text{ða} = 0.5249\ \text{K} \\
2^{-106}\ \text{ergbbe}\cdot\text{s} = \hbar = 1\ \text{ðe}\cdot\text{s} & & 1\ \text{ðm} = 2^{-166}\ \text{qbbm} & & 1\ \text{qbbm} = 2^{166}\ \text{ðm} = 109.8\ \text{g} \\
2^{30}\ \text{fbbl}/\text{s} = c = 1\ \text{ðl}/\text{s} & & 1\ \text{ðl} = 2^{30}\ \text{fbbl} & & 1\ \text{fbbl} = 2^{-20}\ \text{ðl} = 0.2792\ \text{m}
\end{array}$$

$$\begin{array}{lllll}
1\ \text{ergbe}\cdot\text{sbt}/\text{Fbq}^2 = Z_0 = 1\ \text{þe}\cdot\text{þt}/\text{þq}^2 & & 1\ \text{þq} = 2^{-53}\ \text{Fbq} & & 1\ \text{Fbq} = 2^{53}\ \text{þq} = 4.766\ \text{mC} \\
2^{-70}\ \text{ergbe}/{}^\circ\text{ba} = k = 1\ \text{þe}/\text{þa} & & 1\ \text{þa} = 2^{106}\ {}^\circ\text{ba} & & 1\ {}^\circ\text{ba} = 2^{-106}\ \text{þa} = 0.4926\ \text{K} \\
2^{-106}\ \text{ergbe}\cdot\text{sbt} = \hbar = 1\ \text{þe}\cdot\text{þt} & \overset{(3)}{\Rightarrow} & 1\ \text{þm} = 2^{-24}\ \text{qbm} & \Rightarrow & 1\ \text{qbm} = 2^{24}\ \text{þm} = 103.0\ \text{g} \\
2^{30}\ \text{fbl}/\text{sbt} = c = 1\ \text{þl}/\text{þt} & & 1\ \text{þl} = 2^{-112}\ \text{fbl} & & 1\ \text{fbl} = 2^{112}\ \text{þl} = 0.2975\ \text{m} \\
2^{-28}\ \text{dynbf}\cdot\text{fbl}^2/\text{qbm}^2 = \mathbb{G} = 1\ \text{þf}\cdot\text{þl}^2/\text{þm}^2 & & 1\ \text{þt} = 2^{-142}\ \text{sbt} & & 1\ \text{sbt} = 2^{142}\ \text{þt} = 1.0655\ \text{s}
\end{array}$$

$$\therefore \mathbb{G} = 1.135 \times 2^{-28}\ \text{dynbbf}\cdot\text{fbbl}^2/\text{qbbm}^2 \qquad \because 1\ \text{ðq} \equiv 1\ \text{þq} \quad \therefore 1\ \text{Fbbq} \equiv 1\ \text{Fbq}$$

Parity and the ð and þ Units

A parity relation on the power of the scaling factor for $\hbar$ in the to-be-derived unit determines whether the practically sized charge unit is commensurable with the respective ð or þ charge unit. An even power leads to commensurable units, an odd power does not and introduces a factor of the square root of the number base. A parity relation on the sum of powers of scaling factors for $\hbar$, c and $\mathbb{G}$ in the to-be-derived units determines whether the practically sized units of time, distance, mass and temperature are commensurable with their respective ð or þ units. An even sum of powers leads to commensurable units.

CODATA 2018 and Number Bases for Practically Sized Units

exact constants in SI	approximate constants in SI
$e = 160.217\,663\,4 \times 10^{-21}$ C	$Z_0 = 376.730\,313\,668\,\Omega$
$k = 13.806\,49 \times 10^{-24}$ J/K	$\pm 0.000\,000\,057\,\Omega$
$h = 662.607\,015 \times 10^{-36}$ J·s	$G = 66.743\,0 \times 10^{-12}$ N·m²/kg²
$c = 299\,792\,458$ m/s	$\pm 0.001\,5 \times 10^{-12}$ N·m²/kg²

We have chosen to size our derived natural units with a unit of mass somewhat less than a kilogramme and a unit of charge much less than a coulomb. This makes it tricky to express the values of the conversions between systems in a numeric table which directly express the (1) and (3) transformations while avoiding scientific notation for those numbers. Instead, we can state the fundamental constants of nature not in SI, but in the coherent system of second-metre-gramme-kelvin-millicoulomb units by juggling a power of a thousand here or there. After applying our (1) and (3) transformations we get a nicely scaled table of numbers which needs little or no formatting.

Conversion from Coherent Second-Metre-Gramme-Kelvin-Millicoulomb Units

Z_0		0.376 730 313 668		$mC^{-2}\cdot g\cdot m^2\cdot s^{-1}$	
	sexagenary ß, f	denary d	binary b		
k	$30.053\,723\,554\times60^{-12}$	$13.806\,490\,000\times10^{-21}$	$16.299\,826\,406\times2^{-70}$	$K^{-1}\cdot g\cdot m^2\cdot s^{-2}$	
$\hbar$	$10.710\,226\,810\times60^{-18}$	$10.545\,718\,176\times10^{-32}$	$8.555\,703\,025\times2^{-106}$	$g\cdot m^2\cdot s^{-1}$	
c	$0.385\,535\,568\times60^{5}$	$0.299\,792\,458\times10^{9}$	$0.279\,203\,484\times2^{30}$	$m\cdot s^{-1}$	
$\mathbb{G}$	$0.000\,652\,187\times60^{-5}$	$0.000\,838\,717\times10^{-9}$	$0.000\,225\,141\times2^{-28}$	$g^{-1}\cdot m^3\cdot s^{-2}$	
1 Q	5.331 925 227	5.290 817 690	4.765 544 915	mC	
1 Θ	0.356 369 379	0.763 823 258	0.524 895 346	K	
1 M	72.055 936 326	117.336 939 202	109.752 402 408	g	
1 L	0.385 535 568	0.299 792 458	0.279 203 484	m	
1 T	1.000 000 000	1.000 000 000	1.000 000 000	s	(1)
1 Q	5.331 925 227	5.290 817 690	4.765 544 915	mC	
1 Θ	0.393 528 558	0.399 667 433	0.492 628 146	K	
1 M	79.569 318 736	61.396 079 272	103.005 528 350	g	
1 L	0.349 131 133	0.572 947 488	0.297 491 344	m	
1 T	0.905 574 378	1.911 147 106	1.065 500 116	s	(3)
	vicenary ϕ	duodenary ȸ	ternary θ		
k	$9.048\,221\,286\times20^{-16}$	$52.930\,768\,998\times12^{-20}$	$13.596\,229\,613\times3^{-44}$	$K^{-1}\cdot g\cdot m^2\cdot s^{-2}$	
$\hbar$	$1.769\,277\,917\times20^{-24}$	$25.033\,037\,071\times12^{-30}$	$9.776\,878\,706\times3^{-67}$	$g\cdot m^2\cdot s^{-1}$	
c	$0.234\,212\,858\times20^{7}$	$0.697\,221\,442\times12^{8}$	$0.257\,938\,912\times3^{19}$	$m\cdot s^{-1}$	
$\mathbb{G}$	$0.001\,073\,558\times20^{-7}$	$0.004\,327\,597\times12^{-9}$	$0.000\,108\,312\times3^{-17}$	$g^{-1}\cdot m^3\cdot s^{-2}$	
1 Q	2.167 118 926	8.151 574 230	5.094 303 676	mC	
1 Θ	0.195 538 754	0.472 939 229	0.719 087 496	K	
1 M	32.253 332 255	51.495 831 304	146.948 954 583	g	
1 L	0.234 212 858	0.697 221 442	0.257 938 912	m	
1 T	1.000 000 000	1.000 000 000	1.000 000 000	s	(1)
1 Q	2.167 118 926	8.151 574 230	5.094 303 676	mC	
1 Θ	0.119 110 177	0.583 246 314	0.746 681 382	K	
1 M	19.646 745 367	63.506 581 759	152.587 896 577	g	
1 L	0.384 498 551	0.565 358 688	0.248 406 684	m	
1 T	1.641 662 863	0.810 873 926	0.963 044 631	s	(3)